

# 7-1 Lesson Notes

## Focus

**5-Minute Check**  
**Transparency 7-1** Use as a quiz or review of Chapter 6.

**Mathematical Background** notes are available for this lesson on p. 340C.

**How** can the geometric mean be used to view paintings?

*Ask students:*

- What happens if you view the painting at a distance that is less than the geometric mean of the two distances described above? **You are too close to the painting and cannot see all of it at once. Your eyes must move left, right, up, and down to see all the details of the painting.**
- Name some other scenarios where you would be best positioned at a geometric mean to view something. **watching a program on television or a movie in a theatre, viewing a sports event, seeing a production on a stage, using a computer, skimming a book or magazine, etc.**

# 7-1 Geometric Mean

## What You'll Learn

- Find the geometric mean between two numbers.
- Solve problems involving relationships between parts of a right triangle and its altitude to its hypotenuse.

## How can the geometric mean be used to view paintings?

When you look at a painting, you should stand at a distance that allows you to see all of the details in the painting. The distance that creates the best view is the geometric mean of the distance from the top of the painting to eye level and the distance from the bottom of the painting to eye level.



## Vocabulary

- geometric mean

## Study Tip

### Means and Extremes

In the equation  $x^2 = ab$ , the two  $x$ 's in  $x^2$  represent the *means*, and  $a$  and  $b$  represent the *extremes* of the proportion.

**GEOMETRIC MEAN** The geometric mean between two numbers is the positive square root of their product.

## Key Concept

*Geometr*

For two positive numbers  $a$  and  $b$ , the geometric mean is the positive number  $x$  such that the proportion  $a : x = x : b$  is true. This proportion can be written using fractions  $\frac{a}{x} = \frac{x}{b}$  or with cross products as  $x^2 = ab$  or  $x = \sqrt{ab}$ .

## Example 1 Geometric Mean

Find the geometric mean between each pair of numbers.

a. 4 and 9

Let  $x$  represent the geometric mean.

$$\frac{4}{x} = \frac{x}{9} \quad \text{Definition of geometric mean}$$

$$x^2 = 36 \quad \text{Cross products}$$

$$x = \sqrt{36} \quad \text{Take the positive square root of each side.}$$

$$x = 6 \quad \text{Simplify.}$$

b. 6 and 15

$$\frac{6}{x} = \frac{x}{15} \quad \text{Definition of geometric mean}$$

$$x^2 = 90 \quad \text{Cross products}$$

$$x = \sqrt{90} \quad \text{Take the positive square root of each side.}$$

$$x = 3\sqrt{10} \quad \text{Simplify.}$$

$$x \approx 9.5 \quad \text{Use a calculator.}$$

## Resource Manager

### Workbook and Reproducible Masters

#### Chapter 7 Resource Masters

- Study Guide and Intervention, pp. 351–352
- Skills Practice, p. 353
- Practice, p. 354
- Reading to Learn Mathematics, p. 355
- Enrichment, p. 356

*Prerequisite Skills Workbook*, pp. 11–12, 31–32

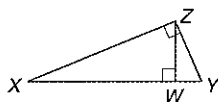
### Transparencies

5-Minute Check Transparency 7-1  
 Answer Key Transparencies

### Technology

Interactive Chalkboard

**ALTITUDE OF A TRIANGLE** Consider right triangle  $XYZ$  with altitude  $\overline{WZ}$  drawn from the right angle  $Z$  to the hypotenuse  $\overline{XY}$ . A special relationship exists for the three right triangles,  $\triangle XYZ$ ,  $\triangle XZW$ , and  $\triangle ZYW$ .



## Geometry Software Investigation

### Right Triangles Formed by the Altitude

Use Geometer's Sketchpad to draw a right triangle with right angle  $Z$ . Draw the altitude  $\overline{ZW}$  from the right angle to the hypotenuse. Explore the relationships between the three right triangles formed.

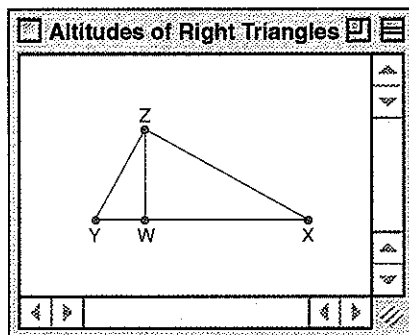
#### Find and Discuss

Find the measures of  $\angle X$ ,  $\angle XZY$ ,  $\angle Y$ ,  $\angle XWZ$ ,  $\angle XZW$ ,  $\angle ZYW$ , and  $\angle ZYW$ . See students' work.

What is the relationship between the measures of  $\angle X$  and  $\angle ZYW$ ? What is the relationship between the measures of  $\angle Y$  and  $\angle XZW$ ? They are equal.

Move point  $Z$  to another position. Describe the relationship between the measures of  $\angle X$  and  $\angle ZYW$  and between the measures of  $\angle Y$  and  $\angle XZW$ . They are equal.

Make a conjecture about  $\triangle XYZ$ ,  $\triangle XZW$ , and  $\triangle ZYW$ . They are similar.

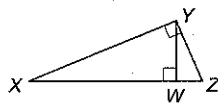


The results of the Geometry Software Investigation suggest the following theorem.

### Theorem 7.1

If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and to each other.

**Example:**  $\triangle XYZ \sim \triangle XWZ \sim \triangle ZYW$



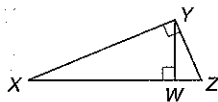
You will prove this theorem in Exercise 45.

By Theorem 7.1, since  $\triangle XWZ \sim \triangle ZYW$ , the corresponding sides are proportional. Thus,  $\frac{XW}{YW} = \frac{YW}{ZW}$ . Notice that  $\overline{XW}$  and  $\overline{ZW}$  are segments of the hypotenuse of the largest triangle.

### Theorem 7.2

The measure of an altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse.

**Example:**  $YW$  is the geometric mean of  $XW$  and  $ZW$ .



You will prove this theorem in Exercise 46.

## Geometry Software Investigation

Students should recognize that if they were given just one angle measure other than the right angle, they would have enough information to find all the other angles represented in the figure. They should also see that given one segment length, they could use proportions to find any of the other segment lengths in the figure as well.

## 2 Teach

### GEOMETRIC MEAN

#### In-Class Example



1 Find the geometric mean between each pair of numbers.

- 2 and 50 10
- 25 and 7  $\approx 13.2$

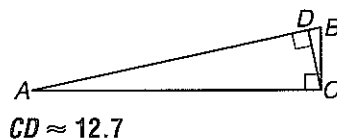
### ALTITUDE OF A TRIANGLE

#### In-Class Example



**Teaching Tip** Remind students that they automatically discard the negative root when finding the altitude or geometric mean because these values represent lengths, and lengths cannot be negative.

2 In  $\triangle ABC$ ,  $BD = 6$  and  $AD = 27$ . Find  $CD$ .



### Interactive Chalkboard

PowerPoint® Presentations

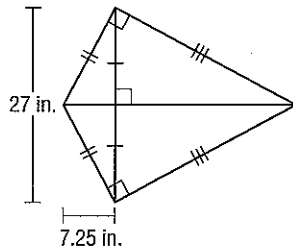
This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

- Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
- Additional, Try These exercises for each example
- The 5-Minute Check Transparencies
- Hot links to Glencoe Online Study Tools

## In-Class Examples

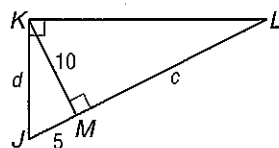
Power Point®

- 3 KITES** Ms. Alspach is constructing a kite for her son. She has to arrange perpendicularly two support rods, the shorter of which is 27 inches long. If she has to place the short rod 7.25 inches from one end of the long rod in order to form two right triangles with the kite fabric, what is the length of the long rod?



$$\approx 32.39 \text{ in.}$$

- 4** Find  $c$  and  $d$  in  $\triangle JKL$ .



$$c = 20; d \approx 11.2$$

### Study Tip

#### Square Roots

Since these numbers represent measures, you can ignore the negative square root value.

### Example 2 Altitude and Segments of the Hypotenuse

In  $\triangle PQR$ ,  $RS = 3$  and  $QS = 14$ . Find  $PS$ .

Let  $x = PS$ .

$$\frac{RS}{PS} = \frac{PS}{QS}$$

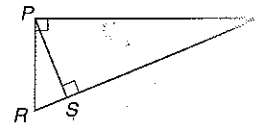
$$\frac{3}{x} = \frac{x}{14} \quad RS = 3, QS = 14, \text{ and } PS = x$$

$$x^2 = 42 \quad \text{Cross products}$$

$$x = \sqrt{42} \quad \text{Take the positive square root of each side.}$$

$$x \approx 6.5 \quad \text{Use a calculator.}$$

$PS$  is about 6.5.



Ratios in right triangles can be used to solve problems.

### Example 3 Altitude and Length of the Hypotenuse

**ARCHITECTURE** Mr. Martinez is designing a walkway that must pass over an elevated train. To find the height of the elevated train, he holds a carpenter square at eye level and sights along the edges from the street to the top of the train. If Mr. Martinez's eye level is 5.5 feet above the street and he is 8.75 feet from the train, find the distance from the street to the top of the train to the nearest tenth.

Draw a diagram. Let  $\overline{YX}$  be the altitude drawn from the right angle of  $\triangle WYZ$ .

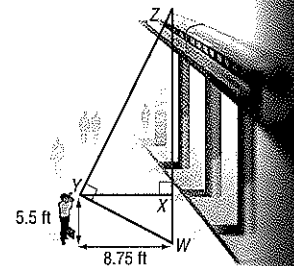
$$\frac{WX}{YX} = \frac{YX}{ZX}$$

$$\frac{5.5}{8.75} = \frac{8.75}{ZX} \quad WX = 5.5 \text{ and } YX = 8.75$$

$$5.5ZX = 76.5625 \quad \text{Cross products}$$

$$ZX \approx 13.9 \quad \text{Divide each side by 5.5.}$$

Mr. Martinez estimates that the elevated train is  $5.5 + 13.9$  or about 19.4 feet high.

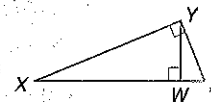


The altitude to the hypotenuse of a right triangle determines another relationship between the segments.

### Theorem 7.3

If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the measure of a leg of the triangle is the geometric mean between the measures of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

$$\text{Example: } \frac{XZ}{XY} = \frac{XY}{XW} \text{ and } \frac{XZ}{YZ} = \frac{ZY}{WZ}$$



You will prove Theorem 7.3 in Exercise 47.

## DAILY INTERVENTION

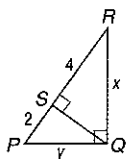
### Differentiated Instruction

**Intrapersonal** Allow students to sit quietly and explore similarities and differences between Theorem 7.2 and Theorem 7.3. Encourage students to use the examples in the book or create their own to reinforce the concepts outlined in these two theorems. Ask students to think and write about why the formulas for geometric mean work for a right triangle with an altitude drawn to its hypotenuse.

### Example 4 Hypotenuse and Segment of Hypotenuse

Find  $x$  and  $y$  in  $\triangle PQR$ .

$PQ$  and  $RQ$  are legs of right triangle  $PQR$ .  
Use Theorem 7.3 to write a proportion for each leg and then solve.



$$\frac{PR}{PQ} = \frac{PQ}{PS}$$

$$\frac{6}{y} = \frac{y}{2}$$

$$y^2 = 12$$

$$y = \sqrt{12}$$

$$y = 2\sqrt{3}$$

$$y \approx 3.5$$

$$PS = 2, PQ = y, PR = 6$$

Cross products

Take the square root.

Simplify.

Use a calculator.

$$\frac{PR}{RQ} = \frac{RQ}{SR}$$

$$\frac{6}{x} = \frac{x}{4}$$

$$x^2 = 24$$

$$x = \sqrt{24}$$

$$x = 2\sqrt{6}$$

$$x \approx 4.9$$

$$RS = 4, RQ = x, PR = 6$$

Cross products

Take the square root.

Simplify.

Use a calculator.

### For Understanding

#### Concept Check

- OPEN ENDED** Find two numbers whose geometric mean is 12.
- Draw and label** a right triangle with an altitude drawn from the right angle. From your drawing, explain the meaning of the *hypotenuse* and the *segment of the hypotenuse adjacent to that leg* in Theorem 7.3. **See margin.**
- FIND THE ERROR**  $\triangle RST$  is a right isosceles triangle. Holly and Ian are finding the measure of altitude  $\overline{SU}$ .

Holly

$$\frac{RS}{SU} = \frac{SU}{RT}$$

$$\frac{9.9}{x} = \frac{x}{14}$$

$$x^2 = 138.6$$

$$x = \sqrt{138.6}$$

$$x \approx 11.8$$

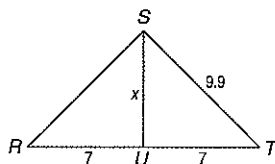
Ian

$$\frac{RU}{SU} = \frac{SU}{UT}$$

$$\frac{7}{x} = \frac{x}{7}$$

$$x^2 = 49$$

$$x = 7$$



Who is correct? Explain your reasoning.

#### Practice

Find the geometric mean between each pair of numbers.

4. 9 and 4

5. 36 and 49

6. 6 and 8

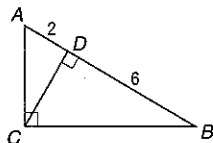
7.  $2\sqrt{2}$  and  $3\sqrt{2}$

$$4\sqrt{3} \approx 6.9$$

$$2\sqrt{3} \approx 3.5$$

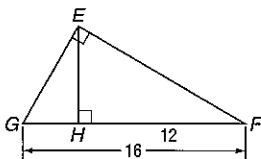
Find the measure of the altitude drawn to the hypotenuse.

8.



$$2\sqrt{3} \approx 3.5$$

9.

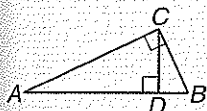


$$4\sqrt{3} \approx 6.9$$

Lesson 7-1 Geometric Mean 345

### Answer

2. For leg  $\overline{CB}$ ,  $\overline{DB}$  is the segment of the hypotenuse that shares an endpoint. Thus, it is the adjacent segment. The same is true for leg  $\overline{AC}$  and segment  $\overline{AD}$ .



## 3 Practice/Apply

### Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 7.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

### DAILY

#### INTERVENTION FIND THE ERROR

Ask students to name the hypotenuse of  $\triangle RST$ , and point out that Ian correctly found the geometric mean between the two segments of the *hypotenuse*, as the rule states in Theorem 7.2.

### About the Exercises...

#### Organization by Objective

- Geometric Mean: 13–20
- Altitude of a Triangle: 21–32

#### Odd/Even Assignments

Exercises 13–32 are structured so that students practice the same concepts whether they are assigned odd or even problems.

**Alert!** Exercise 40 requires the Internet or other research materials.

#### Assignment Guide

**Basic:** 13–31 odd, 35–43 odd, 44, 48–66

**Average:** 13–43 odd, 44, 48–66

**Advanced:** 14–44 even, 45–63 (optional: 64–66)

## Study Guide and Intervention, p. 351 (shown) and p. 352

**Geometric Mean** The geometric mean between two numbers is the square root of their product. For two positive numbers  $a$  and  $b$ , the geometric mean of  $a$  and  $b$  is the positive number  $x$  in the proportion  $\frac{a}{x} = \frac{x}{b}$ . Cross multiplying gives  $x^2 = ab$ , so  $x = \sqrt{ab}$ .

- Example** Find the geometric mean between each pair of numbers.
- a. 12 and 3      b. 8 and 4
- Let  $x$  represent the geometric mean.      Let  $x$  represent the geometric mean.
- $\frac{12}{x} = \frac{x}{3}$        $\frac{8}{x} = \frac{x}{4}$
- $x^2 = 36$        $x^2 = 32$
- $x = \sqrt{36}$  or 6       $x = \sqrt{32}$
- Take the square root of each side.      = 5.7

**Practice** Find the geometric mean between each pair of numbers.

1. 4 and 4      2. 4 and 6       $\sqrt{24} \approx 4.9$
3. 6 and 9       $\sqrt{54} \approx 7.3$       4.  $\frac{1}{2}$  and 2      1
5.  $2\sqrt{3}$  and  $3\sqrt{3}$        $\sqrt{18} \approx 4.2$       6. 4 and 25      10
7.  $\sqrt{5}$  and  $\sqrt{6}$        $18^{\frac{1}{2}} \approx 2.1$       8. 10 and 100       $\sqrt{1000} \approx 31.6$
9.  $\frac{1}{2}$  and  $\frac{1}{4}$        $\sqrt{\frac{1}{8}} \approx 0.4$       10.  $\frac{2\sqrt{2}}{3}$  and  $\frac{3\sqrt{3}}{5}$        $\sqrt{\frac{12}{25}} \approx 0.7$
11. 4 and 16      8      12. 3 and 24       $\sqrt{72} \approx 8.5$

The geometric mean and one extreme are given. Find the other extreme.

13.  $\sqrt{24}$  is the geometric mean between  $a$  and 6. Find  $a$  if  $a = -2$ .      12
14.  $\sqrt{12}$  is the geometric mean between  $a$  and  $b$ . Find  $b$  if  $a = -3$ .      4

Determine whether each statement is *always*, *sometimes*, or *never* true.

15. The geometric mean of two positive numbers is greater than the average of the two numbers.      **never**
16. If the geometric mean of two positive numbers is less than 1, then both of the numbers are less than 1.      **sometimes**

## Skills Practice, p. 353 and Practice, p. 354 (shown)

Find the geometric mean between each pair of numbers to the nearest tenth.

1. 8 and 12      2.  $3\sqrt{7}$  and  $6\sqrt{7}$       3.  $\frac{1}{2}$  and 2
- $\sqrt{96} \approx 9.8$        $\sqrt{126} \approx 11.2$        $\sqrt{\frac{1}{8}} \approx 1.3$

Find the measure of each altitude. State exact answers and answers to the nearest tenth.

4.  $\sqrt{60} \approx 7.7$
5.  $\sqrt{102} \approx 10.1$

Find  $x$ ,  $y$ , and  $z$ .

6.  $\sqrt{184} \approx 13.6$ ;  $\sqrt{248} \approx 15.7$ ;  $\sqrt{713} \approx 26.7$
7.  $\sqrt{114} \approx 10.7$ ;  $\sqrt{150} \approx 12.2$ ;  $\sqrt{475} \approx 21.8$
8. 4.5;  $\sqrt{13} \approx 3.6$ ; 6.5
9. 15;  $\sqrt{300} \approx 17.3$

**10. CIVIL ENGINEERING** An airport, a factory, and a shopping center are at the vertices of a right triangle formed by three highways. The airport and factory are 5.0 miles apart. Their distances from the shopping center are 3.6 miles and 4.8 miles, respectively. A service road will be constructed from the shopping center to the highway that connects the airport and factory. What is the shortest possible length for the service road? Round to the nearest hundredth. 2.88 mi

## Reading to Learn Mathematics, p. 355

ELL

- Pre-Activity** How can the geometric mean be used to view paintings?
- Read the introduction to Lesson 7-1 at the top of page 342 in your textbook.
- What is a disadvantage of standing too close to a painting? **Sample answer:** You don't get a good overall view.
  - What is a disadvantage of standing too far from a painting? **Sample answer:** You can't see all the details in the painting.

### Reading the Lesson

1. In the past, when you have seen the word *mean* in mathematics, it referred to the *average* or *arithmetic mean* of the two numbers.
- a. Complete the following by writing an algebraic expression in each blank.
- If  $a$  and  $b$  are two positive numbers, then the geometric mean between  $a$  and  $b$  is  $\sqrt{ab}$  and their arithmetic mean is  $\frac{a+b}{2}$ .
- b. Explain in words, without using any mathematical symbols, the difference between the geometric mean and the arithmetic mean. **Sample answer:** The geometric mean between two numbers is the square root of their product. The arithmetic mean of two numbers is half their sum.
2. Let  $r$  and  $s$  be two positive numbers. In which of the following equations is  $x$  equal to the geometric mean between  $r$  and  $s$ ? A, C, D, F
- A.  $\frac{r}{x} = \frac{x}{s}$       B.  $\frac{r}{x} = \frac{s}{x}$       C.  $s : x = r : s$       D.  $\frac{r}{x} = \frac{x}{s}$       E.  $\frac{r}{x} = \frac{s}{x}$       F.  $\frac{r}{x} = \frac{x}{s}$
3. Supply the missing words or phrases to complete the statement of each theorem.

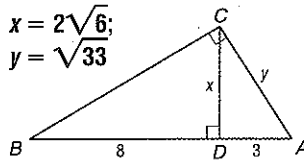
- a. The measure of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the **geometric mean** between the measures of the two segments of the **hypotenuse**.
- b. If the altitude is drawn from the vertex of the **right** angle of a right triangle to its hypotenuse, then the measure of a **leg** of the triangle is the **geometric mean** between the measure of the hypotenuse and the segment of the **hypotenuse** adjacent to that leg.
- c. If the altitude is drawn from the **vertex** of the right angle of a right triangle to its **hypotenuse**, then the two triangles formed are **similar** to the given triangle and to each other.

### Helping You Remember

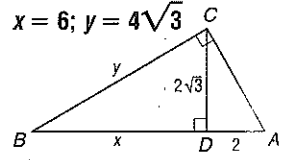
4. A good way to remember a new mathematical concept is to relate it to something you already know. How can the meaning of *mean* in a proportion help you to remember how to find the geometric mean between two numbers? **Sample answer:** Write a proportion in which the two means are equal. This common mean is the geometric mean between the two extremes.

Find  $x$  and  $y$ .

10.  $x = 2\sqrt{6}$ ;  
 $y = \sqrt{33}$

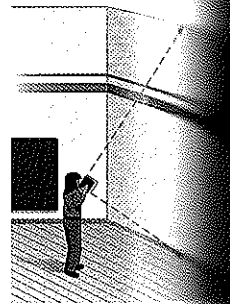


11.  $x = 6$ ;  $y = 4\sqrt{3}$



## Application

12. **DANCES** Khaliah is making a banner for the dance committee. The banner is to be as high as the wall of the gymnasium. To find the height of the wall, Khaliah held a book up to her eyes so that the top and bottom of the wall were in line with the top edge and binding of the cover. If Khaliah's eye level is 5 feet off the ground and she is standing 12 feet from the wall, how high is the wall? **33.8 ft**



★ indicates increased difficulty

## Practice and Apply

### Homework Help

For Exercises	See Examples
13–20	1
21–26	2
27–32	3, 4

### Extra Practice

See page 766.

Find the geometric mean between each pair of numbers.

13. 5 and 6      14. 24 and 25      15.  $\sqrt{45}$  and  $\sqrt{80}$       16.  $\sqrt{28}$  and  $\sqrt{7}$
17.  $\frac{3}{5}$  and 1      18.  $\frac{8\sqrt{3}}{5}$  and  $\frac{6\sqrt{3}}{5}$       19.  $\frac{2\sqrt{2}}{6}$  and  $\frac{5\sqrt{2}}{6}$       20.  $\frac{13}{7}$  and  $\frac{6}{7}$

Find the measure of the altitude drawn to the hypotenuse.

21.  $3\sqrt{5} \approx 6.7$
22. 12
23.  $8\sqrt{2}$
24.  $\sqrt{147} \approx 12.1$
25.  $\sqrt{26} \approx 5.1$
26. 2.5

Find  $x$ ,  $y$ , and  $z$ . 27–32. See margin.

27.  $x$ ,  $y$ ,  $z$
28.  $x$ ,  $y$ ,  $z$
29.  $x$ ,  $y$ ,  $z$
30.  $x$ ,  $y$ ,  $z$
31.  $x$ ,  $y$ ,  $z$
32.  $x$ ,  $y$ ,  $z$

## 346 Chapter 7 Right Triangles and Trigonometry

### Enrichment, p. 356

#### Mathematics and Music

Pythagoras, a Greek philosopher who lived during the sixth century B.C., believed that all nature, beauty, and harmony could be expressed by whole-number relationships. Most people remember Pythagoras for his teachings about right triangles. (The sum of the squares of the legs equals the square of the hypotenuse.) But Pythagoras also discovered relationships between the musical notes of a zither. These relationships can be expressed as ratios.

C	D	E	F	G	A	B	C
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8

When you play a stringed instrument, you produce different notes by placing your finger on different places on a string. This is the result of changing the length of the vibrating part of the string.

The C string can be used to produce F by placing a finger  $\frac{3}{4}$  of the way along the string.



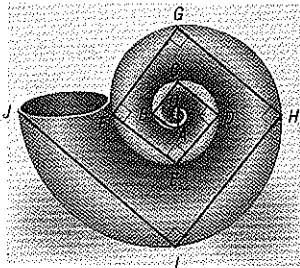
The geometric mean and one extreme are given. Find the other extreme.

- ★ 33.  $\sqrt{17}$  is the geometric mean between  $a$  and  $b$ . Find  $b$  if  $a = 7$ .  $\frac{17}{7}$   
 ★ 34.  $\sqrt{12}$  is the geometric mean between  $x$  and  $y$ . Find  $x$  if  $y = \sqrt{3}$ .  $4\sqrt{3} \approx 6.9$

Determine whether each statement is *always*, *sometimes*, or *never* true.

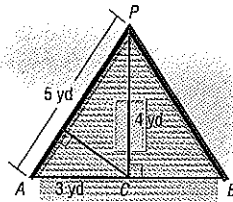
35. The geometric mean for consecutive positive integers is the average of the two numbers. **never**  
 36. The geometric mean for two perfect squares is a positive integer. **always**  
 37. The geometric mean for two positive integers is another integer. **sometimes**  
 38. The measure of the altitude of a triangle is the geometric mean between the measures of the segments of the side it intersects. **sometimes**

- 39. **BIOLOGY** The shape of the shell of a chambered nautilus can be modeled by a geometric mean. Consider the sequence of segments  $\overline{OA}$ ,  $\overline{OB}$ ,  $\overline{OC}$ ,  $\overline{OD}$ ,  $\overline{OE}$ ,  $\overline{OF}$ ,  $\overline{OG}$ ,  $\overline{OH}$ ,  $\overline{OI}$ , and  $\overline{OJ}$ . The length of each of these segments is the geometric mean between the lengths of the preceding segment and the succeeding segment. Explain this relationship. (*Hint*: Consider  $\triangle FGH$ .) **See margin.**



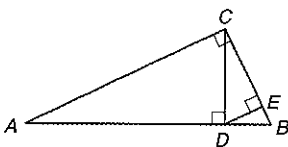
40. **RESEARCH** Refer to the information at the left. Use the Internet or other resource to write a brief description of the golden ratio.

41. **CONSTRUCTION** In the United States, most building codes limit the steepness of the slope of a roof to  $\frac{4}{3}$ , as shown at the right. A builder wants to put a support brace from point  $C$  perpendicular to  $\overline{AP}$ . Find the length of the brace. **2.4 yd**



**Soccer** For Exercises 42 and 43, refer to the graphic.

42. Find the geometric mean between the number of players from Indiana and North Carolina.  $\sqrt{70} \approx 8.4$   
 43. Are there two schools whose geometric mean is the same as the geometric mean between UCLA and Clemson? If so, which schools? **yes; Indiana and Virginia**  
 44. **CRITICAL THINKING** Find the exact value of  $DE$ , given  $AD = 12$  and  $BD = 4$ .  $2\sqrt{3}$



By Ellen J. Horrow and Adrienne Lewis, USA TODAY

## Answers

39.  $\triangle FGH$  is a right triangle.  $\overline{OG}$  is the altitude from the vertex of the right angle to the hypotenuse of that triangle. So, by Theorem 7.2,  $\overline{OG}$  is the geometric mean between  $\overline{OF}$  and  $\overline{OH}$ , and so on.

## Answers (p. 346)

27.  $x = 2\sqrt{22} \approx 9.4$ ;  $y = \sqrt{33} \approx 5.7$ ;  $z = 2\sqrt{6} \approx 4.9$   
 28.  $x = \frac{50}{3}$ ;  $y = 10$ ;  $z = \frac{40}{3}$   
 29.  $x = \frac{40}{3}$ ;  $y = \frac{5}{3}$ ;  $z = 10\sqrt{2} \approx 14.1$   
 30.  $x = 2\sqrt{21} \approx 9.2$ ;  $y = 21$ ;  $z = 25$   
 31.  $x = 6\sqrt{6} \approx 14.7$ ;  $y = 6\sqrt{42} \approx 38.9$ ;  $z = 36\sqrt{7} \approx 95.2$   
 32.  $x = 4\sqrt{6} \approx 9.8$ ;  $y = 4\sqrt{2} \approx 5.7$ ;  $z = 4\sqrt{3} \approx 6.9$