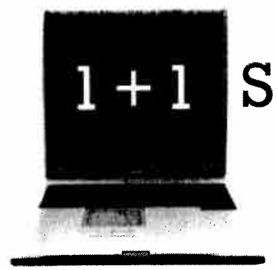
I think both examples are reasonable early-generation math ed video games. But the screen images in Figure 2 remind me of the ornithopter in Figure 1. (Though to be fair, unlike the ornithopters those video games do manage to get off the ground.) Let's face it, in real life you are rather unlikely to find yourself face to face with a person or a monster who has a math problem written on their chest, or a situation in which a math formula suddenly appears before your eyes, hovering in the air. Symbolic expressions are the way people do math when they are working with paper and pencil. They look out of place in a game world. And the reason they look out of place in that environment is because they *are* out of place.

Video game worlds are not paper-and-pencil symbolic representations; they are imaginary *worlds*. They are meant to be lived in and experienced. (Not all video games have worlds. But any game is played in an environment, in part created by the game, that affects the action. If you don't believe this, pay particular attention next time you see someone playing a casual game like *Bejeweled* on a smartphone. I'll discuss the special case of those kinds of games in the final chapter.)

Putting symbolic expressions in a math ed game environment is to confuse mathematical thinking with its static, symbolic representation on a sheet of paper, just as the early aviators confused flying with the one particular representation of flying which they had observed. To build truly successful math ed video games, we have to separate the activity—a form of thinking—from its familiar representation in terms of symbolic expressions.

Mathematical symbols were introduced to do mathematics first in the sand, then on parchment and slate, and still later on paper and blackboards. *Video games provide an entirely different representational medium.* As a dynamic medium, video games are far better suited in many ways to representing and doing middle-school mathematics than are symbolic expressions on a page. We need to get beyond thinking of video games as an environment that delivers traditional pedagogy—a new canvas on which to pour symbols—and see them as an entirely new medium to represent mathematics. That is in my view the single most important message of this book.

Now that I have provided you with a rough road map of where we are heading, I'll explain to you why video games are the way to go. The story begins twenty years ago in the streets of South America.



Street Smarts

The young woman walks up to a stall piled high with coconuts. Behind it stands a young boy of around 12 years of age, who is taking care of the stall while his parents have stepped away for a moment. It's hot and there is a lot of noisy activity in the market, one of several in the Brazilian city of Recife.

"How much is one coconut?" the woman asks.

"Thirty-five," the boy replies with a smile.

"I'd like ten. How much is that?"

The boy pauses for a moment before replying. Thinking out loud, he says: Three will be 105; with three more, that will be 210. (Pause) I need four more. That is ... (pause) 315 ... I think it is 350."

Though the boy gets the answer right, the woman can't help but wonder why he did not use the simple rule that to multiply by 10 you just add a 0, so ten coconuts at 35 Cruzeiros each will cost Cr$350.

A short while later, at another stall, again staffed by a young boy, this one about 14 years old, the woman makes a purchase that requires the child to subtract Cr$75 from Cr$243. The boy calculates out loud:

"You just give me the two hundred. I'll give you twenty-five back. Plus the forty-three that you have, the hundred and forty-three, that's one hundred and sixty-eight."

If you were the shopper, faced with a young child at a stall in a noisy, busy South American street market who calculated your change in that fashion, you might suspect that the young salesman was trying to pull a fast one. In fact, his answer is perfectly correct. In a moment I'll examine what he is doing.

Incidentally, you may think it strange that a book about using video games in education begins the pedagogy section with a discussion about street markets. You may even be tempted to skip the rest of this chapter and look for the "new stuff" about video games. However, what you will find in this chapter is crucial for the entire remainder of the book, and sets the stage for everything else.

Returning to those two marketplace exchanges, I am not making them up. They are taken verbatim from a report written some years ago by three researchers, Terezinha Nunes (the "young woman" in my story), then at the University of London, England, now at Oxford University), and Analucia Dias Schliemann and David William Carraher of the Federal University of Pernambuco in Recife, Brazil.[1] The three researchers went out into the street markets of Recife with concealed tape recorders, posing as ordinary market shoppers. They sought out stalls being staffed by young children between 8 and 14 years of age. At each stall, they presented the young stallholder with a transaction designed to test a particular arithmetical skill. The purpose of the research was to compare traditional instruction (which all the young market traders had received in school since the age of six) with learned practices in context.[2]

How well did our two young sellers do? Let's start with the coconut seller, who did not appear to know the rule that to multiply by 10 you simply add a 0—so 35 becomes 350. It turns out that this simple trick isn't the only mathematical fact he did not know. Despite spending six years in school, he had almost no mathematical knowledge at all in the traditional sense. When given school-type tasks in a school-type setting he performed badly. What arithmetical skills he had were self-taught at his market stall. Here is how he solved the problem.

Because he often found himself selling coconuts in groups of two or three, he needed to be able to compute the cost of two or three coconuts; that is, he needed to know the values 2 × 35 = 70 and 3 × 35 = 105. Faced with a highly unusual request for ten coconuts—how was the woman going to carry ten coconuts and what was she going to do with them, by the way?—the young boy proceeded like this. First, he split the 10 into groups he could handle, namely 3 + 3 + 3 + 1. Arithmetically, he was now faced with the determining the sum 105 + 105 + 105 + 35. He did this in stages. With a little effort, he first calculated 105 + 105 = 210. Then he computed 210 + 105 = 315. Finally, he worked out 315 + 35 = 350. Altogether quite an impressive performance for a twelve-year-old who supposedly "couldn't do math"!

[1] T. Nunes, A. Schliemann, and D. Carraher, *Street Mathematics and School Mathematics*, Cambridge University Press, Cambridge, UK, 1993.

[2] A similar study I'll mention later was described in J. Lave, *Cognition in Practice: Mind, Mathematics and Culture in Everyday Life*, Cambridge University Press, Cambridge, UK, 1988. More generally, see J. S. Brown, A. Collins, and P. Duguid, "Situated Cognition and the Culture of Learning," Educational Researcher 18 (1), 1989, pp. 32–41.

What about the second boy, the one with the whiz-bang answer to 243 – 75? First, it's clear from what he went on to say that his first sentence was meant to be "You just give me the one hundred." For what he was doing was splitting up the 243 into 100 + 100 + 43. He put the 43 and one of the 100s to one side and subtracted the 75 from the remaining 100. That's something he could do easily: 100 – 75 = 25. (Presumably it's a result he had seen so often that he knew it by heart.) Then he added back the 43 and the 100. To do this, he first computed 100 + 43 = 143 and then calculated 25 + 143 = 168. That last step was still a challenging addition, of course, and the boy struggled with it. But in the end he got the right answer.

In essence, what his overall method did was change the challenging subtraction problem 243 – 75 into the addition problem 143 + 25, by subtracting 100 from 243 and adding 100 to 75. The final addition was not an easy one, but his method worked because, like most people, he found addition much easier than subtraction.

Pretty remarkable, don't you think? But there's more. Posing as customers was just the first stage of the study Nunes and her colleagues carried out. About a week after they had "tested" the children at their stalls, they went back to the subjects and asked each of them to take a pencil-and-paper test that included exactly the same arithmetic problems that had been presented to them in the context of purchases the week before.

The investigators were careful to give this second test in as non-threatening a way as possible. It was administered in a one-on-one setting, either at the original location or in the subject's home, and included both straightforward arithmetic questions presented in written form and verbally presented word problems in the form of sales transactions of the same kind the children carried out at their stalls. The subjects were provided with paper and pencil, and were asked to write their answer and whatever working they wished to put down. They were also asked to speak their reasoning aloud as they went along.

Although the children's arithmetic had been close to flawless when they were at their market stalls—just over 98% correct—they averaged only 74% when presented with market stall word problems requiring the same arithmetic, and a staggeringly low 37% when the same problems were presented to them in the form of a straightforward symbolic arithmetic test.

The performance of our young coconut seller was typical. One of the questions he had been asked at his market stall, when he was selling coconuts costing Cr$35 each, was: "I'm going to take four coconuts. How much is that?" The boy replied: "There will be one hundred five, plus thirty, that's one thirty-five . . . one coconut is thirty-five . . . that is . . . one forty." Let's take a look at this solution. Just as he had in the transaction I described first, the boy began by breaking the problem up into simpler ones; in this case, three coconuts plus one coconut. This enabled him to start out with the fact he knew, namely that three coconuts cost Cr$105. Then, to add on the cost of the fourth coconut, he first rounded the cost of a coconut to Cr$30 and added that amount to give Cr$135. While not verbalizing the next step precisely, he noted that the "correction factor" for the rounding was Cr$5, and added in that correction factor to give the correct answer of Cr$140.

On the formal arithmetic test, the boy was asked to calculate 35 × 4. He worked mentally, vocalizing each step as the researcher had requested, but the only thing he wrote down was the answer. Here is what he said: "Four times five is twenty, carry the two; two plus three is five, times four is twenty." He then wrote down 200 as his answer. Despite the fact that, numerically, it was the same problem he had answered correctly at his market stall, he got it wrong. If you follow what he said, it's clear what he was doing and why he went wrong. In trying to carry out the standard right-to-left school method for multiplication, he added the carry from the units-column multiplication (5 × 4) *before* performing the tens-column multiplication rather than afterwards, which is the correct way. He did, however, keep track of the positions the various digits should occupy, writing the (correct) 0 from the first multiplication after the (incorrect) 20 from the second, to give his answer 200.

The same thing happened with another child stallholder, this time a girl of nine. When a researcher approached the child at her coconut stall and asked, "I'll take three coconuts. How much is that?" the young seller replied, "Forty, eighty, one twenty." With one coconut costing Cr$40, her technique was to keep adding 40 until she reached the correct number of additions. On the school-like arithmetic test, the same girl was presented with the multiplication 40 × 3. Her answer was 70. Her explanation of how she arrives at that answer was: "Lower the zero; four and three is seven."

Despite the fact that she had no trouble operating a stall in a noisy, busy street market, the young girl's recollections of the standard arithmetical procedures she had been taught in school were mired in confusion. How bad was her confusion? The same girl, upon being asked for 12 lemons priced at Cr$5 each, separated them out two at a time, saying as she did so, "Ten, twenty, thirty, forty, fifty, sixty." But when she was presented with the problem 12 × 5 on the test—numerically the very same computation—she first lowered the 2, then the 5, and then the 1, giving the answer 152.

A similar degree of confusion about school arithmetic was exhibited by another child street-seller who had no trouble with a subtraction task when it had arisen at the market stall, but went badly awry when presented with the equivalent addition on the school-like written test. At the market stall, where the boy had been selling coconuts for Cr$40 each, the customer paid with a Cr$500 bill, and said, "I'll take two coconuts. What do I get back?" "Eighty, ninety, one hundred, four twenty," the boy replied.

On the test, the child was presented with the addition 420 + 80. He gave the answer 130, apparently proceeding as follows: add 8 to 2 to give 10; carry the 1; add 1 (the carry), 4, and 8, to give 13; write down the final 0 in the units column to give 130. Eventually, with some prodding by the researcher, the boy was able to reach the right answer—by ignoring the pencil and paper and using a counting on method.

A similar outcome arose in another case, after a subject had failed to solve the division problem: 100/4. She first tried to divide 1 by 4, then tried to divide 0 by 4, and then gave up, claiming that it was not possible. Prodded by the researcher, she replied: "See, in my head I can do it . . . Divide by two, that's fifty. Then divide by two, that's twenty-five."In other words, she used the fact that dividing by 4 can be achieved by dividing by 2 twice in succession—together with her ability to halve the numbers 100 and 50.

In case after case, Nunes and her colleagues obtained the same results. The children were absolute number wizards when they were at their market stalls, but virtual dunces when presented with the same arithmetic problems presented in a typical school format. The researchers were so impressed—and intrigued—by the children's market stall performances that they gave it a special name: they called it *street mathematics*.

Street Mathematics

Street mathematics is the mathematics that people develop for themselves, when they need it. It is not restricted to young market traders in Brazil, and you can find it in other locations besides the streets. For instance, you can find it in the United States, as schoolteacher James Herndon described in his 1971 book *How to Survive in Your Native Land.*[3]

Herndon recounts how, on one occasion, he was teaching a junior high school class of children who had all essentially failed in the school system. At one point, he discovered that one of the students had a well-paid, regular job scoring for a local bowling league, a task that required fast, accurate, and complicated arithmetic. (Have you ever seen the scoring system in bowling?) Seeing a golden opportunity to motivate this student to do well in class, Herndon created a set of "bowling score problems" and gave them to the boy. The attempt was a complete failure. During evenings in the bowling alley, the boy could keep accurate track of eight different bowling scores at once. But he could not answer the simplest scoring question when it was presented to him in the classroom. In Herndon's own words, "The brilliant league scorer couldn't decide whether two strikes and a third frame of eight amounted to eighteen or twenty-eight or whether it was one hundred eight and a half."

[3] Simon and Schuster, New York, 1971.

Herndon observed similar failure when he tried to reach other students in the class by presenting them with problems of the very kind they solved with ease outside the classroom. For example, to a girl who admitted she never had any trouble shopping for clothes, he gave the problem: "If you buy a pair of shoes costing $10.95, how much change do you get from a twenty?" The girl answered "$400.15," and wanted Herndon to tell her if it was right!

Since both the Recife children and Herndon's students demonstrated that they could handle arithmetic in the appropriate context, when the numbers meant something to them, and when the consequences mattered to them, it seems clear that meaning and motivation play major roles in our ability to do arithmetic. (These may not be the only factors at work. Social and cultural circumstances may also play an important attitudinal role, and I'll discuss these and other issues affecting performance in due course.)

Achievement level was not the only difference between street mathematics and school mathematics. The transcriptions of the verbal market stall exchanges also showed that the children's street methods of computation were different from those taught at school. Yet the school methods are taught in part because they are supposed to be easier! Indeed, for anyone who masters both methods, those taught in school *are* easier—just compare the method our first subject used to compute 10 × 35 with the schoolroom method for solving the same problem. Moreover, the school methods are much more general than some of the "quick tricks" picked up on the job, and therefore can be applied much more widely. Seeking powerful, efficient, and more general methods is an important part of mathematics.

Why did the people who used street mathematics seem to ignore the standard methods? Intrigued by this question, Nunes and her colleagues set out to examine the methods used by the child stallholders. The researchers' approach was to determine the difference between the children's abilities in mental (or oral) arithmetic and written arithmetic, when both were measured under test conditions. The children never performed as well during formal testing as they did when at work at their stalls. Nunes and her colleagues wanted to know if there was a measurable difference between the two ways of doing arithmetic on a test, and sought to explore how the *methods* of street mathematics and school arithmetic differed.

The group of children that Nunes and her colleagues tested consisted of 16 students, including boys and girls. All were in the third grade at school, where they had been taught the standard procedures for addition, subtraction, multiplication, and division. Because many children in Brazil have to repeat the same grade level two or more times, the ages of the children ranged from 9 to 15. The older children had not only more years instruction at school arithmetic, they had also spent longer working in the street market. The subjects were given three kinds of problems: simulated sales transactions of the kind they were familiar with in the market, word (or story) problems, and straightforward computational arithmetic problems. In all but one category, the children performed better at mental arithmetic than they did with pencil and paper. In most cases, the differences were dramatic.

In the case of addition, for the simulated sales questions, the children averaged 67% correct orally and 75% correct on the written test. This was the only case where their pencil-and-paper results were better than their oral answers (i.e., the answers they obtained by working in their heads without the aid of pencil and paper). For the addition word problems they averaged 83% correct orally and just 62% written. For the straightforward computation questions, they got a perfect 100% orally compared with a significantly lower 79% in writing.

For subtraction, the difference between their oral performance and their written performance was striking for all three kinds of questions. In the simulated sales they averaged a so-so 57% correct orally (far less than when calculating change at their stalls) and a mere 22% correct in writing. For the word problems, the figures were a moderately good 69% orally and that same low 22% in writing. For the computation problems, their performances were 60% correct orally—not too bad—but a miserable 14% in writing.

For multiplication, the corresponding figures were a comfortable 89% correct orally and a disappointing 50% correct in writing for the simulated sales, 64% orally and 50% in writing for the word problems, and a perfect 100% orally against a poor 39% in writing for the computation problems.

When it came to division, the results were extremely poor. The children averaged 50% correct orally on all three kinds of problems, but they had clearly failed to master the schoolroom method of division. When asked to answer the questions using pencil and paper, they scored 0% correct on the simulated sales and the word problems, and got just 7% correct on the straightforward division questions. In short, the children could not do division under any sort of test conditions.

Clearly, the children were much better at mental arithmetic than they were at applying the paper-and-pencil methods they had been taught at school. (Presumably the same will be true of anyone who makes regular use of numbers and basic arithmetic in their lives.) But there is still the question of how they were achieving their much greater success in oral arithmetic compared to written arithmetic. Since they appeared unable to use the methods they had been taught in school, just *how* were these children solving the problems when they worked them in their heads?

You get some idea of the children's methods—and hence a first indication that street mathematics is something very different from school arithmetic—when you look at the transcripts of what the children actually said as they were working out the problems mentally. Their words reveal that they were using some sophisticated manipulations of numbers. For example, when faced with computing 200 – 35, one child proceeded like this: "If it were thirty, then the result would be seventy. But it's thirty-five. So it's sixty-five. One hundred sixty-five."

Here is what he was doing. First he split the 200 into 100 + 100. (He did not vocalize this step, but it's clear from what came after that this is what he did.) He put one 100 to one side and set out to compute 100 – 35. To do this, he first rounded off 35 to 30, and computed 100 – 30. This he could do easily; the answer is 70. Then he corrected for the rounding by subtracting the 5 he had ignored; 70 – 5 = 65. Finally, he added the 100 he had put to one side at the beginning: 65 + 100 = 165.

Here is one more example, this time involving division. As we saw earlier, most of the children had significant difficulty with division when working orally and failed completely when trying to use the school-taught procedure. The problem was to calculate 75/5, asked as a question about sharing 75 marbles among 5 children. One child said, "If you give ten marbles to each, that's fifty. There are twenty-five left over. To distribute to five boys, twenty-five, that's hard. . . . That's five more for each. Fifteen each." Absolutely right! The child began by "rounding" 75 to 50 and solving the simpler problem 50/5, for which he had no trouble computing the answer 10. He appeared to know that as a fact, which is why he performed the initial rounding down from 75 to 50. The rounding left 25 marbles still to distribute. He found this difficult; he did not know the answer to 25/5. But after a bit of thought he figured it out: 25/5 = 5. Now all he had to do was add that 5 to his previous result of 10 to give his final answer, 15.

Grown-Ups Too

It's not just children that exhibit a huge disparity between the math they are able to do in the everyday world and their poor performance when presented with a "math test." I'll describe one particular study of adults because it illustrates well an additional finding of such research that I believe will be important in the design of any video game intended to develop mathematical thinking.

In the early 1980s, the anthropologist Jean Lave carried out a study called the Adult Math Project (AMP).[4] Currently a faculty member in the Department of Education at the University of California at Berkeley, Lave was at the University of California at Irvine at the time of this study. The subjects she studied were ordinary people in Southern California, shopping in a supermarket.

Lave and her colleagues followed the shoppers—all selected because they were price conscious—around the store, observing them, taking copious notes,

[4] J. Lave, *Cognition in Practice: Mind, Mathematics, and Culture in Everyday Life*. Cambridge University Press, Cambridge, UK, 1988.

occasionally asking them to explain their reasoning out loud as they went about their shopping, and sometimes asking for explanations just after the transaction had been completed. Of course, this procedure is highly contrived. The very presence of an observer changes the experience of shopping. Thus, to some extent the study was not really one of people in their normal, everyday activities, but it was close enough for the purpose of the study, and moreover anthropologists have developed ways of going about such work so as to minimize any influences of their presence on their subjects' behavior.

Each of the researchers spent a total of about 40 hours with each of her or his subjects, including time spent interviewing them to determine their backgrounds (education, occupation, etc.). Though most of the shoppers were women, there were some men in the group. However, the researchers noticed no difference in the mathematical performances of the men and the women in the supermarket, so gender did not seem to be a significant factor.

Out of a total of approximately 800 individual purchases that the shoppers made in the course of the study, just over 200 involved some arithmetic—which the researchers defined to be "an occasion on which a shopper associated two or more numbers with one or more arithmetical operations: addition, subtraction, multiplication, or division." The shoppers varied enormously in the frequency with which they used mathematics. One shopper used none whatsoever, while three of the subjects performed calculations in making over half their purchases.

Among the arithmetical techniques that the researchers observed shoppers performing were estimation, rounding (e.g., to the nearest dollar or the nearest dollar and a half), and left-to-right calculation (as opposed to the right-to-left calculation taught in school). What seemed to be absent, however, were most of the techniques the shoppers had been taught in school. Lave and her colleagues set out to investigate where the school math had gone. In order to compare the shoppers' arithmetical performance in the supermarket with their ability to do "school math," the researchers designed a test to determine the latter. Again, the results were fascinating. Despite the significant efforts the researchers made to persuade the subjects that this was not like a school test, rather that its purpose was purely to ascertain what arithmetical ability they had retained since school and with nothing at stake, the shoppers treated it as if it were indeed a school test. They approached it in "math test mode," with all of the accompanying stresses and emotions.

Perhaps the shoppers' reaction was to be expected. After all, the "math test" did have all the elements of a typical school arithmetic test, including questions involving whole numbers (both positive and negative), fractions, decimals, addition, subtraction, multiplication, and division. On the other hand, the problems were designed to test the same arithmetical skills that the researchers had observed the shoppers using in the supermarket. For instance, having observed working out the problems mentally. Their words reveal that they were using some sophisticated manipulations of numbers. For example, when faced with computing 200 – 35, one child proceeded like this: "If it were thirty, then the result would be seventy. But it's thirty-five. So it's sixty-five. One hundred sixty-five."

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One obvious difference was that the subjects approached the test questions as requiring a precise calculation, but they were much more likely to use estimation in tackling their real-life equivalents (although often with considerable accuracy). Another difference was that the shoppers in the supermarket were *not* using the arithmetical skills they learned in school. Rather, they were solving the problems another way. This conclusion is supported by the fact that performance on the test was higher the longer subjects had studied math at school and the more recently they had finished school, whereas neither length of schooling nor the time since schooling had any measurable effect on how well they did in the supermarket. Thus, if they teach anything, traditional school math classes seem to teach people how to perform on school math tests. They do not teach them how to solve real-life problems that involve math. A few schools have tried to address this issue by adopting a different approach that encourages students to think about problems and try to develop their own methods, but standard testing makes that a very difficult goal to achieve.

What seemed to make the biggest difference in test performance was the kind of test the shoppers were asked to take and the manner in which the questions were presented. This was shown by a further test the AMP researchers put the subjects through: a shopping simulation.

In their homes, the subjects were presented with simulated best-buy shopping problems, based on the very same best-buy problems the researchers had observed them resolve in the supermarket. In some of these simulations, the subjects were presented with actual cans, bottles, jars, and packets of various items taken from the supermarket and asked to decide which to buy among competing brands. In other simulations they were presented with the price and quantity information printed on cards. In this simulation, which was clearly a kind of "test" situation but with the questions of a shopping nature as opposed to school-like "math questions," the subjects scored an average of 93% correct.

The fact that the simulation was done in each subject's home, carried out by the researcher who had accompanied the subject on the shopping trip, seems to have been a significant factor. Not only did the subjects not view it as a "math test,"

they managed to approach most of the questions using the same mental resources they had used in the store. The researchers went to some effort to achieve this, such as giving the subjects the questions verbally in a conversational fashion and making frequent references to the actual shopping expedition the two had gone on together.

The importance of setting up the shopping simulation test this way becomes clear when you compare the AMP results with those from another shopping simulation test carried out by Deanna Kuhn.[5] Kuhn set up a table outside a Southern Californian supermarket, stopped customers about to enter to do their shopping, and asked them to calculate which of two bottles of garlic powder was the better buy, the 1.25 oz. bottle for 41 cents or the 2.37 oz. bottle for 77 cents, and similarly for two bottles of deodorant, one costing $1.36 for 8 oz., and the other $2.11 for 12 oz. The subjects were given a pencil and paper on which they could do their work.

The results were very different from those obtained in the AMP shopping simulation. Only 20% of the 50 shoppers who agreed to take the test were able to solve the garlic powder question, and not a great many more—just 32%—could solve the deodorant question. The enormous difference between the results observed in the AMP and Kuhn test procedures is almost certainly due to the way the subjects approached the two simulations. In the AMP simulation, the shoppers seemed to understand that they were to imagine they were actually shopping, whereas Kuhn's subjects seemed to view it as "taking a test." In fact, Kuhn's results were very similar to those obtained in the school-like tests administered to the AMP subjects. This confirms that you can carry out the test outside a supermarket and phrase the questions in terms of shopping, even to the point of presenting the subjects with actual items taken from the supermarket shelves, but if the subjects view it as a "math test," that is how they will approach it. As a result, they will struggle to use their long-forgotten—and possibly never fully understood—school math procedures. And more often than not, they will fail. These findings are of crucial importance in designing any video game intended to help students acquire mathematical thinking.

[5] Noel Capon and Deanna Kuhn, "Logical Reasoning in the Supermarket: Adult Females' Use of a Proportional Reasoning Strategy in an Everyday Context", *Developmental Psychology*, Volume 15, Issue 4, July 1979, pp. 450–452.