Milan Sherman and Charity Cayton

Using Appropriate Tools Strategically for Instruction

Students' ability to use appropriate tools strategically is an important skill of mathematically proficient students (SMP 5, CCSSI 2010, p. 7). A parallel practice for teachers is using appropriate tools strategically for mathematics instruction. An important element of this practice is that the use of technology depends on the goals of instruction. A carpenter does not start with a hammer and then decides what to build. Rather, he starts with a goal in mind, such as a bookshelf, and chooses tools necessary to complete the project. Likewise, a teacher first considers mathematical goals and then decides which tools may be most effective in accomplishing them.

Once a teacher has identified mathematical goals, high-level tasks provide opportunities for students to think critically about mathematics (Stein and Smith 1998). Further, technology can support goals related to students' mathematical thinking and behaviors (NCTM 2000; CCSSI 2010). In particular, interactive geometry software, or IGS (known also by the trademarked term *dynamic geometry,* or DGS) support learning important mathematics (Hollebrands and Dove 2011) and students' mathematical thinking (Sherman 2014; Cayton 2012). This led us to ask the following question: What do high-level tasks using technology look like? In this article, we share a research-based framework for critically evaluating and revising interactive geometry tasks to support students' mathematical thinking. We demonstrate how to use this framework by analyzing a task from a geometry textbook and discussing several ways the task could be revised.

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Department editors
Michael Weiss, *mweiss@msu.edu, Michigan State University, East Lansing, MI; and* ***Dana Cox,*** *dana.cox@miamioh.edu, Miami University, Oxford, OH*

The task shown in **figure 1** guides students to construct and manipulate a circle and two intersecting chords to discover that the products of the lengths of the segments formed by the intersection are always equal (HSG.C.A.2, CCSSI 2010). Consider the following questions:

- How essential is the use of technology in achieving the primary goals of the task?
- How does the use of technology contribute to the goals of the task in a way that would be difficult to achieve without it?
- How does the use of technology support students' mathematical thinking?

A FRAMEWORK FOR EVALUATING AND REVISING TECHNOLOGY TASKS

Our framework was inspired by two studies conducted separately by the authors. Sherman (2014) examined the use of technology by secondary school mathematics teachers, with the goal of observing how the use of technology was related to students' mathematical thinking. Cayton (2012) examined teachers' implementation of preconstructed interactive tasks in Algebra 1 classes equipped with one-to-one computing resources to study the influence of design and implementation of tasks on cognitive demand. The common issue addressed by these studies is how interactive geometry systems support students' mathematical thinking. We integrated our findings to develop a framework (see **fig. 2**) for examining the potential of an interactive task to encourage students' high-level thinking with respect to mathematical goals and for suggesting ways a task might be revised to do this more effectively.

Our framework was informed, in part, by Sinclair's design principles (2003) related to how the sketch—that is, the technological representation of mathematical objects—and associated prompts depends on the goal of the task. Cayton (2012) found that when interactive tasks adhered to design principles described by Sinclair (2003), the potential cognitive demand was high. Creation of the framework began by interpreting these principles to identify three overarching goals (see **fig. 2**, col. 1) for students' mathematical activity while using interactive geometry systems.

One design principle that Sinclair (2003) articulates is that when a statement prompts action from students, the sketch should provide affordances for students to perform necessary actions. We associated this design principle with a goal of making mathematically meaningful observations, including looking for invariant relationships—that is, properties that remain constant under a variety of conditions. A second design principle relates to the goal of mathematical exploration. Questions that invite exploration are open-ended, so the sketch must provide alternative paths. The key feature of this goal is how interactive technology is used to support students with respect to what and how they explore. We related a third design principle to the mathematical goal of students making, testing, and revising conjectures. Within this goal, the sketch must support experimentation to unmask any confusion or false conjectures by providing feedback to guide students' thinking.

Although a given task may address more than one of these goals, we treat each as discrete in the sense that a teacher will first identify the main goals of a given task for students' thinking and then consider how the use of interactive geometry systems supports that goal, independently of the other goals.

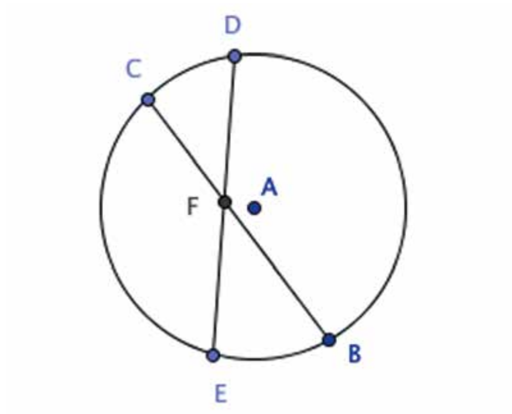
After identifying these overarching goals, we considered the role that technology plays within each goal. The metaphors of *amplifier* and *reorganizer* (Pea 1985) distinguish two ways that technology may be used to support students' thinking. As an amplifier, technology makes a task more efficient by performing computations and generating representations quickly and accurately, but the nature of what students think about is not essentially changed. As a reorganizer, technology has the capability to transform students' activity, supporting a shift in students' thinking to something that would be difficult or impossible to achieve without it. The focus of students' thinking using technology in such tasks might be on noticing patterns and making and testing conjectures. Sherman (2014) found that the use of technology as a reorganizer was associated with high-level tasks. We integrated these ideas into a framework to evaluate and revise the use of technology in a given task according to how technology use might vary, depending on mathematical goals for the lesson.

For each goal (in a separate row of **fig. 2**), a question guides the evaluation of the role of technology, and the response describes the use of technology as an amplifier or a reorganizer. These goals are not necessarily mutually exclusive but are listed separately because of the varying roles that technology plays. We demonstrate how the framework, applied to the task in **figure 1**, can be used to evaluate and revise a task to support students' high-level thinking.

INTERSECTING CHORDS USING GEOGEBRA

In this lab, you will use GeoGebra to explore chords and tangents. When two chords intersect in a circle, the segments formed by their intersection have a special relationship.

1. Use the **Circle with Center through Point** tool to create a circle *A*.
2. Use the **Segment** tool to draw two intersecting chords. Label them *BC* and *DE*.
3. Use the **Intersect** tool to create the point of intersection. Label it *F*.



4. Measure the lengths of *DF*, *EF*, *BF*, and *CF* using the **Distance or Length** tool. Record the length of each segment in the chart below.
5. Complete the last two columns of the chart by multiplying the indicated segments' measures and writing their products in the corresponding blanks.
6. Drag your segments to change the length of your chords and record the new values in row 2 of the chart (but make sure that they are still intersecting). Repeat this step to fill in row 3 of the chart.
7. Using your chart as a guide, make a conjecture about the lengths of the segments formed by intersecting chords of a circle.

DF	*EF*	*BF*	*CF*	*DF* · *EF*	*BF* · *CF*

Source: Adapted from *Saxon Geometry* (2011)

Fig. 1 How does the use of technology contribute to the goals of the task in a way that would be difficult to achieve without it?

EVALUATING A TASK

To distinguish the use of technology as an amplifier or a reorganizer, both the

		Technology Used As	
		Amplifier Students could achieve the same goal without the technology.	**Reorganizer** The mathematical goal of the task would be difficult to achieve without IGS.
Goals	**Question**	**Use of Design Principles**	
Make mathematically meaningful observations; look for invariant relationships	Do the sketch and prompts use the dynamic affordances of IGS in a way that would be difficult or impossible to replicate without it?	Students create multiple static examples, either by construction or dragging, and reason from those static examples. For example, students are prompted to make observations or generalizations based on a table or static measurements without reference to the sketch.	The sketch allows for continuous dragging, and students are guided to examine measurements or relationships dynamically. Students are required to make or explain observations or generalizations dynamically in terms of the sketch.
Mathematical exploration; use appropriate tools strategically	How does technology support mathematical exploration?	Sketch and prompts guide students to investigate the same example or set of examples to explore mathematical connections or invariances. Freedom with respect to dragging does not provide alternative paths if students are all investigating the same example.	Sketch and prompts allow students to explore their individual observations of mathematical concepts, connections, or invariances within the sketch. The sketch supports students' mathematical exploration by providing alternate paths.
Make and test conjectures; modify thinking; foster curiosity	Does the sketch provide feedback? Do the prompts encourage or require students to use feedback?	Sketch is limited by restrictive construction or does not provide feedback to allow students to explore their conjectures. Prompts do not explicitly guide students to test conjectures.	Sketch provides feedback or allows students to test and refine conjectures. Prompts explicitly guide students to use the sketch to test conjectures.

Fig. 2 A framework helps teachers analyze the use of technology with respect to the goals of meaningful observation, exploration, and conjecture.

sketch and the written prompts of the task are considered in relation to the guiding question. Descriptions in the framework help evaluate the task and may suggest revisions for using technology as a reorganizer of student thinking.

Mathematically Meaningful Observations

Analysis related to this first goal follows the top row in the framework. The dynamic capabilities may be used as an amplifier to generate static examples, otherwise accessible, more quickly. The technology encourages students to drag continuously and make observations. Observing how objects change in relation to one another in real time is difficult to replicate outside the interactive system environment and thus constitutes a reorganizer of students' thinking.

In the Intersecting Chords task, students generate static examples, record them in a table, and use the table to look for patterns and make conjectures. Students are prompted to reason from the table, rather than from the sketch. Such a task could be completed without technology using a compass and straightedge. Thus, with regard to the goal of mathematical observation in the first row of the framework, we classify this task as an amplifier. One way to revise the task is to prompt students to examine $DF \cdot EF = BF \cdot CF$ dynamically within the sketch through continuous dragging, instead of a table of static measurements (see **fig. 3**; see also the online component with this article at **http://www.nctm.org/mt**).

Further, observing this relationship dynamically draws attention to the invariant relationships that are the focal point of the task. As Kaput (1992) notes, "One very important aspect of mathematical thinking is the abstraction of invariance. But, of course, to recognize invariance—to see what stays the same—one must have variation" (p. 525). In this example, there are two ways to vary the sketch to see the invariant relationship. First, for a circle with fixed radius, students can drag the chords, changing the lengths and noticing that the products of the segment lengths vary but remain equal to one another. Second, students can vary the size of the circle while leaving the position of the chords fixed, again noticing that as the segment lengths of the chords change with the size of the circle, the products of those segment lengths remain equal to one another. Dragging continuously supports students' exploration of this invariance and provides greater access to this idea. In the original task, prompts 5, 6, and 7 can be revised to use the technology as a reorganizer:

5. In the spreadsheet view, input the products $DF \cdot EF$ and $BF \cdot CF$ in separate rows.

6. Drag the chords by moving the points on the circle. As you drag, what do you notice about these products?
7. Resize the circle by dragging the point on the edge of the circle that you used to create the circle. As you change the size of the circle, what do you notice about the products?

Mathematical Exploration

In the revision described above, the task asks students to focus on a specific mathematical relationship between segments of intersecting chords. The modified tasks use technology as an amplifier with respect to mathematical exploration (see **fig. 2**, row 2) because all students are guided to investigate the same segments and products. However, revision could make use of technology in a way that is more open-ended. The original task could be recast as a reorganizer along the second dimension this way:

5. Explore different ratios and products that you can form from these measurements. Are there any combinations that always remain equal to one another?

Here, students are given freedom to explore segment lengths and their products or ratios, setting up the possibility that some students will discover the equal products property, equal ratios, or both. Thus, the technology provides different pathways to discovering equivalent mathematical relationships, and the prompts are open-ended enough to allow students to explore those pathways.

Making and Testing Conjectures

Students engaging in mathematical investigations may ask, "Is this right?" but by answering this question teachers might lower the cognitive demand of the task for students and reinforce the teacher as the locus of mathematical authority in the classroom (Stein et al. 2009; Reinhart 2000). Feedback from interactive geometry systems supports student independence in assessing their own thinking in a way that may be difficult to replicate without them, thus establishing a reorganizer related to conjecture. This dimension of the framework has two components.

Students rarely make use of information provided by interactive systems to test their conjectures independently (Sherman 2012). Prompting students to check and modify their conjectures can help students develop this habit. Further, students may be more likely to reason about and question the validity of conjectures that they themselves have generated rather than those presented to them as potential theorems (Glass and Deckert 2001). Asking students to look for counterexamples and revise their conjectures reinforces a stance of inquiry versus confirming known results.

Adding a prompt that guides students to look for a counterexample or to find conditions under which their conjecture does not hold can enhance the task. For example, we could append the following prompt to the original task (see **fig. 1**):

8. Specify the conditions under which your conjecture holds. Use the sketch to consider lots of examples, including extreme cases. Modify your conjecture, if necessary. Then use the sketch to gather evidence to support your conjecture.

This prompt encourages students to test and refine their conjectures about the products of the lengths of the segments. For example, students might drag the chords so that their intersection is near or on the edge of the circle. They might also drag the chords so that they no longer intersect or extend the segments to lines to explore intersection points outside the circle. Such prompts encourage students to generalize their observations, make use of feedback provided by the interactive software to test their thinking, and modify it, if necessary. At a minimum, the result should be a better-formulated conjecture. Rather than asking the teacher, students are directed to gather evidence to answer their question, "Is this right?"

We note that this prompt lies at the intersection of interactive geometry systems and proof and that there is a danger of students accepting multiple examples as a proof (Chazan 1993). Doing so may be especially tempting in an environment in which multiple examples are easy to generate (Glass and Deckert 2001). Consequently, it is important to regularly include tasks that do not lead students to view the results of their exploration as a foregone conclusion. The Pentagon task (Zbiek 1996) is an example of this type of task in which "plausible-but-false conjectures are more useful than ever" (p. 89). Not every task needs to lead to "plausible-but-false conjectures" to achieve the goal

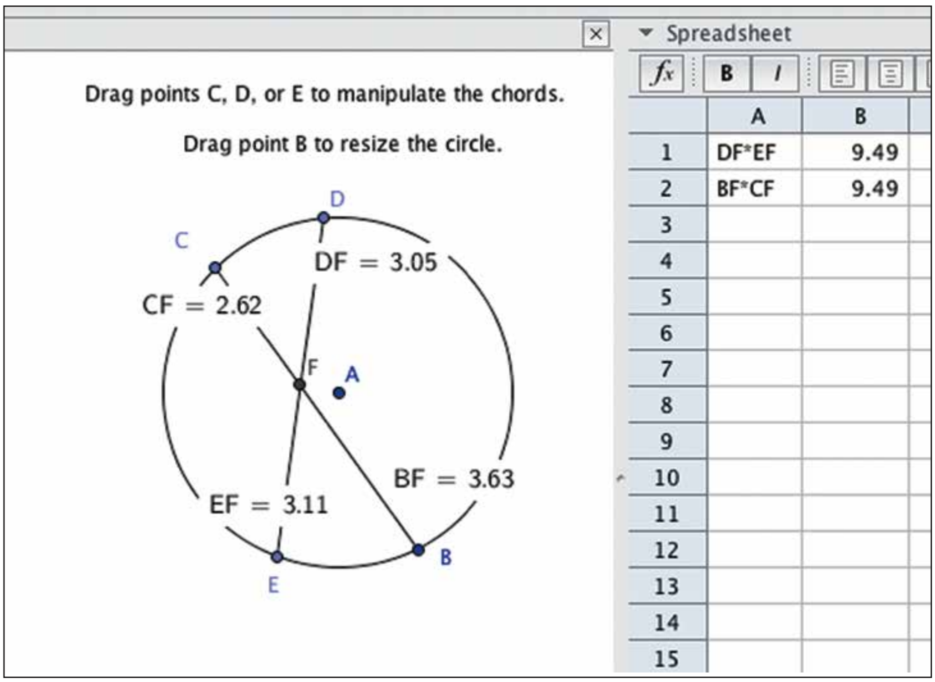


Fig. 3 Dragging continuously allows students to observe invariant products.

of conjecture using technology, but such tasks keep students honest in their exploration and make their search for proof (which sometimes includes counterexamples) sincere.

REVISING TO ADDRESS GOALS

The framework is not intended to serve as a cumulative measure. We would not call a task a reorganizer because it fits the description of a reorganizer on at least two of the three dimensions of the framework. Each row addresses discrete but not necessarily mutually exclusive goals. Thus, the revised tasks described here do not depict a progression of improvement but, rather, separate tasks that address different goals. Nonetheless, a teacher might combine revisions illustrated here to address multiple goals for student thinking. Further, there are many ways to revise a task to better accomplish a given goal; the revisions suggested here are only one way to do that.

We encourage readers not to get too caught up in classifying a task "correctly"; that is not the point of the framework. Also, it is possible to achieve certain goals using technology as an amplifier. Using technology as a reorganizer provides a more mathematically robust experience that supports students' high-level thinking. As such, we view the framework as a tool for assessing how the use of technology supports the goals of instruction and suggesting ways in which a task might be revised to accomplish certain goals more effectively. In this way, the framework can support teachers in using appropriate tools strategically for mathematics instruction.

REFERENCES

Cayton, Charity. 2012. "Examining the Cognitive Demand of Tasks in Three Technology-Intensive High School Algebra 1 Classrooms." In *Proceedings of the Thirty-Fourth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education-North America Chapter*, edited by Laura Van Zoest, Jane-Jane Lo, and James L. Kratky, pp. 865–68. Kalamazoo, MI: Academic Press.

Chazan, Daniel. 1993. "High School Geometry Students' Justification for Their Views of Empirical Evidence and Mathematical Proof." *Educational Studies in Mathematics* 24 (4): 359–87.

Common Core State Standards Initiative (CCSI). 2010. Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/the-standards/mathematics.

Glass, Brad, and Walter Deckert. 2001. "Making Better Use of Computer Tools in Geometry." *Mathematics Teacher* 94 (3): 224–29.

Hollebrands, Karen F., and Anthony Dove. 2011. "Technology as a Tool for Creating and Reasoning about Geometry Task." In *Focus in High School Mathematics: Technology to Support Reasoning and Sense Making*, edited by Tom P. Dick and Karen F. Hollebrands, pp. 33–52. Reston, VA: National Council of Teachers of Mathematics.

Kaput, James J. 1992. "Technology and Mathematics Education." In *Handbook of Research on Mathematics Teaching and Learning*, edited by Douglas A. Grouws, pp. 515–56. New York: Macmillan.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Pea, Roy D. 1985. "Beyond Amplification: Using the Computer to Reorganize Mental Functioning." *Educational Psychologist* 20 (4): 167–82.

Reinhart, Steven. 2000. "Never Say Anything a Kid Can Say." *Mathematics Teaching in the Middle School* 5 (8): 478–83.

Saxon Publishers. 2011. *Saxon Geometry.* Austin, TX: Houghton Mifflin Harcourt.

Sherman, Milan F. 2012. "Supporting Students' Mathematical Thinking during Technology-Enhanced Investigations Using DGS." In *Technology in Mathematics Education: Contemporary Issues,* edited by Dragana Martinovic, Douglas McDougall, and Zekeriya Karadag, pp. 147–80. Santa Rosa, CA: Informing Science Institute.

———. 2014. "The Role of Technology in Supporting Students' Mathematical Thinking: Extending the Metaphors of Amplifier and Reorganizer." *Contemporary Issues in Technology and Teacher Education* 14 (3): 220–46.

Sinclair, Margaret P. 2003. "Some Implications of the Results of a Case Study for the Design of Pre-Constructed, Dynamic Geometry Sketches and Accompanying Materials." *Educational Studies in Mathematics* 52 (3): 289–317.

Stein, Mary Kay, and Margaret S. Smith. 1998. "Mathematical Tasks as a Framework for Reflection: From Research to Practice." *Mathematics Teaching in the Middle School* 3 (4): 268–75.

Stein, Mary Kay, Margaret S. Smith, Marjorie A. Henningsen, and Edward A. Silver. 2009. *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development*. 2nd ed. New York: Teachers College Press.

Zbiek, Rose Mary. 1996. "The Pentagon Problem: Geometric Reasoning with Technology." *Mathematics Teacher* 89 (1): 86–90.

MILAN SHERMAN, milan.sherman@drake.edu, is an assistant professor in the Mathematics and Computer Science Department at Drake University in Des Moines, Iowa. He is interested in the learning and teaching of algebra, the use of technology to support students' mathematical thinking, and mathematics teacher education. **CHARITY CAYTON,** caytonc@ecu.edu, is an assistant professor in the Department of Mathematics, Science, and Instructional Technology Education at East Carolina University in Greenville, North Carolina. She is interested in the design and implementation of technological tasks in high school mathematics classrooms, with a focus on classroom interactions and discourse.

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For a GeoGebra file of figure 3, go to the online component for this print article at **nctm.org/mt**.